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DILATIONS

A **dilation** is a transformation that changes the SIZE of a figure but not its SHAPE. It uses a scale factor to ENLARGE or reduce a figure.

A **pre-image** is the original figure. (BEFORE)

An **image** is the figure after a transformation. (AFTER)

A **scale factor (k)** is used to change the magnification of the given figure.

$$(x,y) \rightarrow (kx,ky)$$

what scale factor does:

Original figure:

scale factor = 1

Scale Factor >1:

size increased

0< Scale Factor< 1

$(\frac{1}{2}, .75, .1)$

size decreases

Scale Factor =-1

negative scale factor flips the image upside-down.

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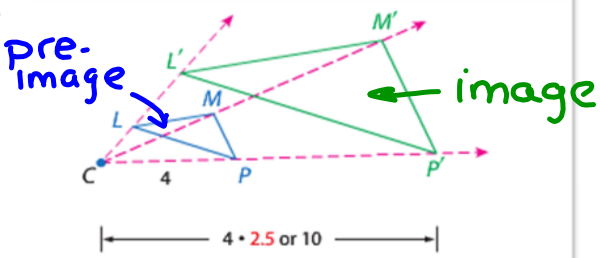
1 Draw Dilations

A dilation or *scaling* is a similarity transformation that enlarges or reduces a figure proportionally with respect to a *center* point and a *scale* factor.

KeyConcept Dilation

A dilation with center C and positive scale factor k, $k \neq 1$, is a function that maps a point P in a figure to its image such that

- if point P and C coincide, then the image and preimage are the same point, or
- if point P is not the center of dilation, then P' lies on $\overrightarrow{CP}$ and $CP' = k(CP)$.



$\triangle L'M'P'$ is the image of $\triangle LMP$ under a dilation with center C and scale factor 2.5.

2 Dilations in the Coordinate Plane

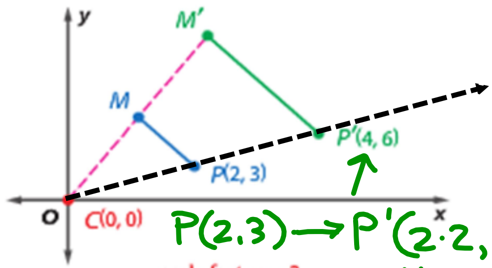
You can use the following rules to find the image of a figure after a dilation centered at the origin.

KeyConcept Dilations in the Coordinate Plane

Words To find the coordinates of an image after a dilation centered at the origin, multiply the x- and y-coordinates of each point on the preimage by the scale factor of the dilation, k.

Symbols $(x, y) \rightarrow (kx, ky)$

Example



$P(2,3) \rightarrow P'(2\cdot 2, 2\cdot 3)$

$(k\cdot x, k\cdot y)$

$k=2$

Transformations:

r = Reflection

R = Rotation

< > or T = Translation

D = Dilation

$D_k(x,y) \rightarrow (kx, ky)$

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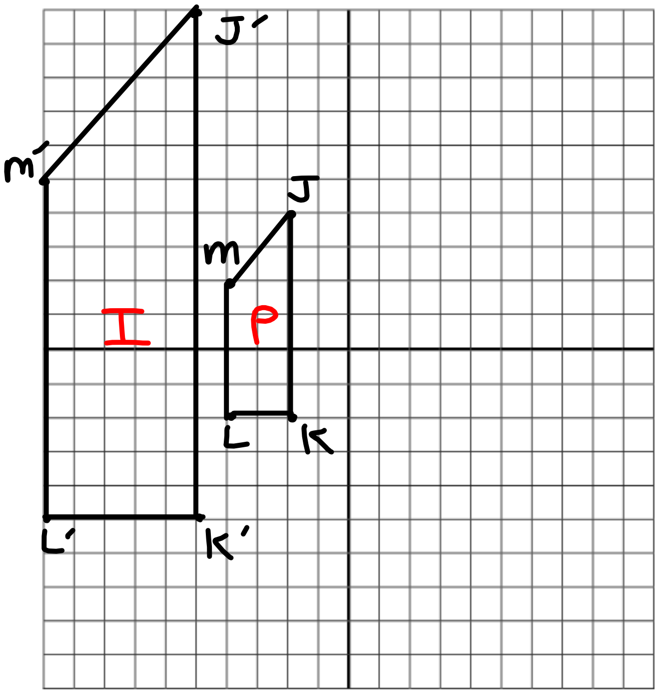
Example 1:

Quadrilateral *JKLM* has vertices *J*(−2, 4), *K*(−2, −2), *L*(−4, −2), and *M*(−4, 2). Graph the image of *JKLM* after a dilation centered at the origin with a scale factor of 2.5.



$D_{2.5}\ (x,y) \to (2.5x, 2.5y)$

$J(-2,4) \to J'(2.5\cdot -2,\ 2.5\cdot 4)$

$J'(-5,10)$

$K(-2,-2) \to K'(2.5\cdot -2,\ 2.5\cdot -2)$

$K'(-5,-5)$

$L(-4,-2) \to L'(2.5\cdot -4,\ 2.5\cdot -2)$

$L'(-10,-5)$

$M(-4,2) \to M'(2.5\cdot -4,\ 2.5\cdot 2)$

$M'(-10,5)$

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EXAMPLES 2-3: Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

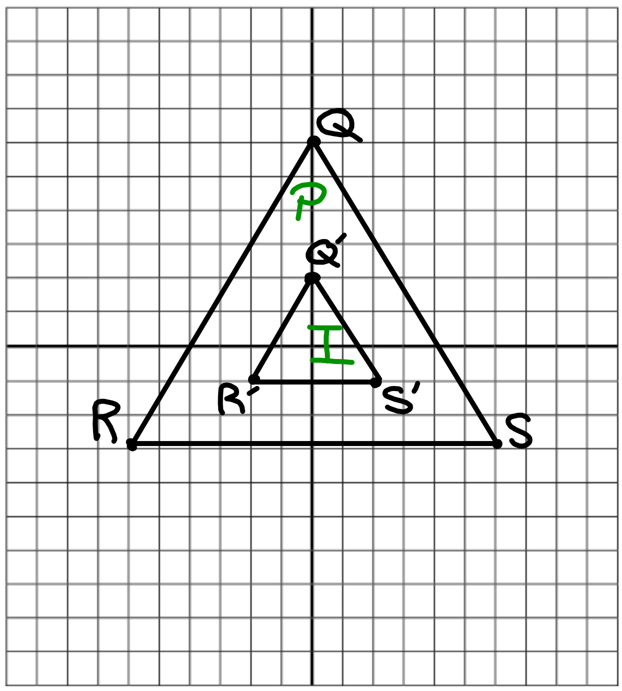
2. Q(0, 6), R(-6, -3), S(6, -3); $k = \frac{1}{3}$

$$D_{\frac{1}{3}}(x, y) \longrightarrow \left(\tfrac{1}{3}x, \tfrac{1}{3}y\right) = \left(\tfrac{x}{3}, \tfrac{y}{3}\right)$$

$$Q(0,6) \rightarrow Q'\left(\tfrac{0}{3}, \tfrac{6}{3}\right) = (0,2)$$

$$R(-6,-3) \rightarrow R'\left(\tfrac{-6}{3}, \tfrac{-3}{3}\right) = (-2,-1)$$

$$S(6,-3) \rightarrow S'\left(\tfrac{6}{3}, \tfrac{-3}{3}\right) = (2,-1)$$



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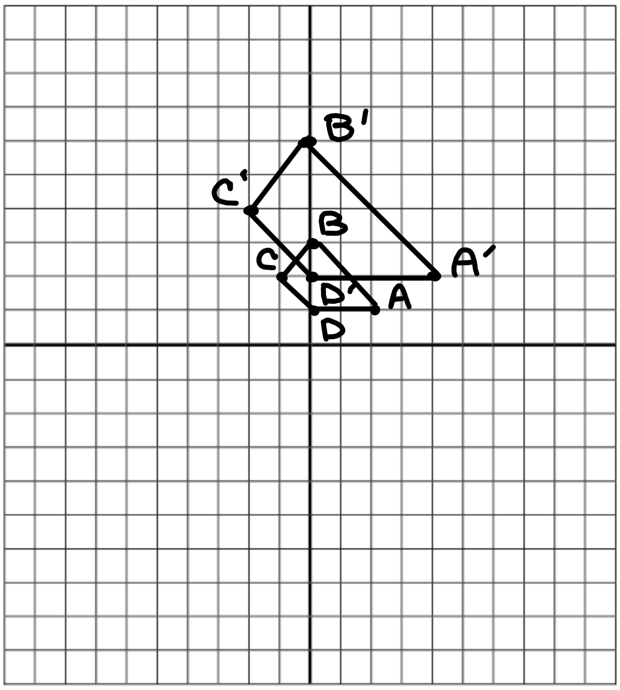
3. A(2, 1), B(0, 3), C(-1, 2), D(0, 1); k = 2.



$D_2 (x,y) \to (2x, 2y)$

$A(2,1) \to A'(2\cdot 2, 2\cdot 1) = (4,2)$

$B(0,3) \to B'(2\cdot 0, 2\cdot 3) = (0,6)$

$C(-1,2) \to C'(2\cdot -1, 2\cdot 2) = (-2,4)$

$D(0,1) \to D'(2\cdot 0, 2\cdot 1) = (0,2)$